pressed as

$$\frac{4(1+\nu)\lambda}{\pi^2 Eh} \sum_{m,n \text{ large}}^{\infty} \left\{\frac{1}{m^2 + n^2\lambda^2}\right\} \times \left\{1 - \frac{(1+\nu)m^2}{2(m^2+\lambda^2 n^2)}\right\} \cos^2(m\pi\bar{x}/L)$$

where $\lambda = L/\pi a$, which is greater than the corresponding sum

$$\frac{4(1+\nu)\lambda^3}{\pi^2 Eh} \sum_{m,n \text{ large}}^{\infty} \left\{\frac{n^2}{(m^2 + n^2\lambda^2)^2}\right\} \cos^2(m\pi\bar{x}/L)$$

An application of the integral test to this latter sum reveals that it diverges. Thus the double Fourier series representation for $\hat{u}$ also diverges. One might have expected this since, for the problem of an in-plane force at a point of an infinite plate, as considered in Ref. 5, the displacement in the direction of the force at the point of application of the force is found to be infinite. Since $\hat{u}$ is infinite, the modal representation for $\hat{u}$ will thus diverge. For fixed ω_c, each one of the infinite number of terms, for which $\omega_j^2 \gg \omega_c^2$, in the modal representation for $\tilde{u}$ assumes the same form as the corresponding term in the divergent modal representation for $\hat{u}$. This then implies the divergence of the modal representation for $\tilde{u}$ and hence of

$$[u]_{\substack{\theta=0\\ x=\bar{x}}}$$

These difficulties may be removed by distributing the attached mass over a small area of the shell.

References

[1] Ojalvo, I. U. and Newman, M., "Natural Vibrations of a Stiffened Pressurized Cylinder with an Attached Mass," *AIAA Journal*, Vol. 5, No. 6, June 1967, pp. 1139–1146.

[2] Bisplinghoff, R. L., Ashly, H., and Halfman, R. L., *Aeroelasticity*, Chap. X, Addison-Wesley, Reading, Mass., 1957, p. 635.

[3] Bushnell, D., "Dynamic Response of Two-Layered Cylindrical Shells to Time Dependent Loads," *AIAA Journal*, Vol. 3, No. 9, Sept. 1965, pp. 1698–1703.

[4] Bijlaard, P. P., "Stresses From Local Loadings in Cylindrical Pressure Vessels," *Transactions of the American Society of Mechanical Engineers*, Vol. 77, 1955, pp. 805–816.

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Reply by Authors to R. D. Smith

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The preceding comments are unfortunate since they are either stated without proof (mathematically speaking), or loosely worded. In addition, the discusser has failed to appreciate a key point in our analysis, i.e., only the unweighted "breathing" modes are employed as trial functions for the weighted cylinder. His three main criticisms are summarized below and then negated individually: 1) The assumption of vanishing axial restraint employed in the analysis is not applicable to the ring-stiffened shells tested. 2) A basic discrepancy exists between the experimental results and the analytical predictions for cylinder 1. 3) The expression for axial displacement [Eq. (29) of the original paper[3]] diverges as the number of modal functions that employed ($\bar{n}$) becomes infinite.

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Since the discusser has attempted a more quantitative argument to support item 3, we choose to deal with it first. He has contended that divergence of our for the

$$[u]_{\substack{\theta=0\\ x=\bar{x}}}$$

weighted cylinder depends upon divergence of his next to last series. We agree that his series divergences, but actually it does not represent the series obtained when only the unweighted cylinder "breathing" modes are employed, as was stated in the paragraph following Eq. (22) of the subject paper.[3] Our approach is similar to that used successfully by Reissner[1] and later by Fung, Sechler, and Kaplan,[2] i.e., only the lowest root of the cubic frequency equation is considered for each nodal pattern. The other two roots are considerably higher than the smallest one and correspond to predominately tangential modes. It is demonstrated in Ref. 2 that for $k > 2$ and $a/h > 100$, the frequencies of the tangential modes are over 60 times larger than the corresponding radial ("breathing") frequencies.

Consistent with this simplification, the contribution to $\hat{u}$ for large wave numbers is actually (using discusser's notation)

$$\frac{(1-\nu^2)(1+\nu)^2\lambda^{11}}{2\pi^2 Eh(\beta^2)(1+\beta)^2} \times \sum_{m,n \text{ large}}^{\infty} \left\{\frac{m^2 n^4 \cos^2(m\pi\bar{x}/L)}{[m^2 + \lambda^2 n^2/(1+\beta)](m^2+\lambda^2 n^2)^6}\right\}$$

where $\beta = h^2/(12a^2)$.
This series is rapidly convergent and therefore obviates the need of 1) "distributing the attached mass over a small area of the shell," and 2) including an axial rigid body mode.

Turning to item 1, the discusser's statement that

$$|u|]_{\text{adjacent bays}} \ll |u|]_{\text{excited bay}}$$

which he has simply stated without proof, does not follow from the experimental observations that

$$|w|]_{\text{adjacent bays}} \ll |w|]_{\text{excited bay}}$$

Further, the experimental evidence for cylinders 1 through 4 inclusive (see Table 2 of our paper[3]) indicates that the axial forces were not sufficiently "appreciable" to affect the rather close agreement with our theoretical results. Thus, it appears that the method of test cylinder fabrication was effective in permitting axial warping across the intermediate rings and at the bulkhead joints.

With regard to item 2, where the discusser refers to ". . . forces exerted by the attached mass . . . act(ing) through displacements of zero amplitude, . . .," it should be evident that the addition of a concentrated mass at a nodal point does not produce any inertia force or "perturbation" of that uncoupled mode, since the mass is motionless. Therefore, a weighted system frequency can equal an unweighted frequency with the mass not driving the shell at resonance.

Finally, both the analytical and experimental evidence tend to contradict the notion that a mass addition merely causes a downward "perturbation" of the unweighted frequencies. Rather, it has been shown that surprisingly small weight additions can create a highly localized response, causing a drastic change in the fundamental mode-shape and a large reduction in fundamental frequency.

References

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[2] Fung, Y. C., Sechler, E. E., and Kaplan, A., "On the Vibration of Thin Cylindrical Shells under Internal Pressure," *Journal of Aeronautical Sciences*, Vol. 24, 1967, pp. 650–660.

[3] Ojalvo, I. U. and Newman, M., "Natural Vibrations of a Stiffened Pressurized Cylinder with an Attached Mass," *AIAA Journal*, Vol. 5, No. 6, June 1967, pp. 1139–1146.